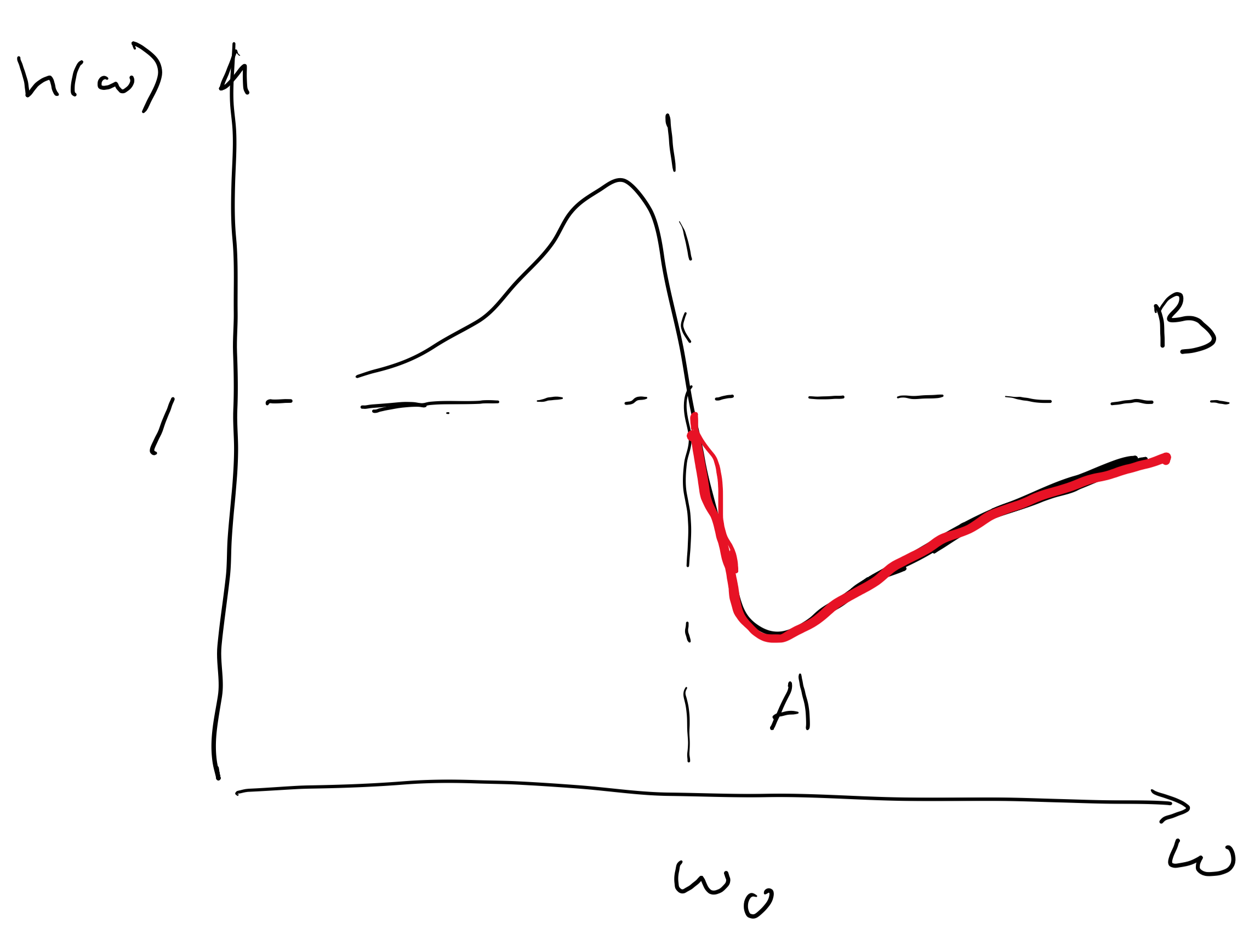


Let's come back to dispersion curve:



Since at the positions
A/B $n < 1 \Rightarrow$ phase speed
 $v > c$.

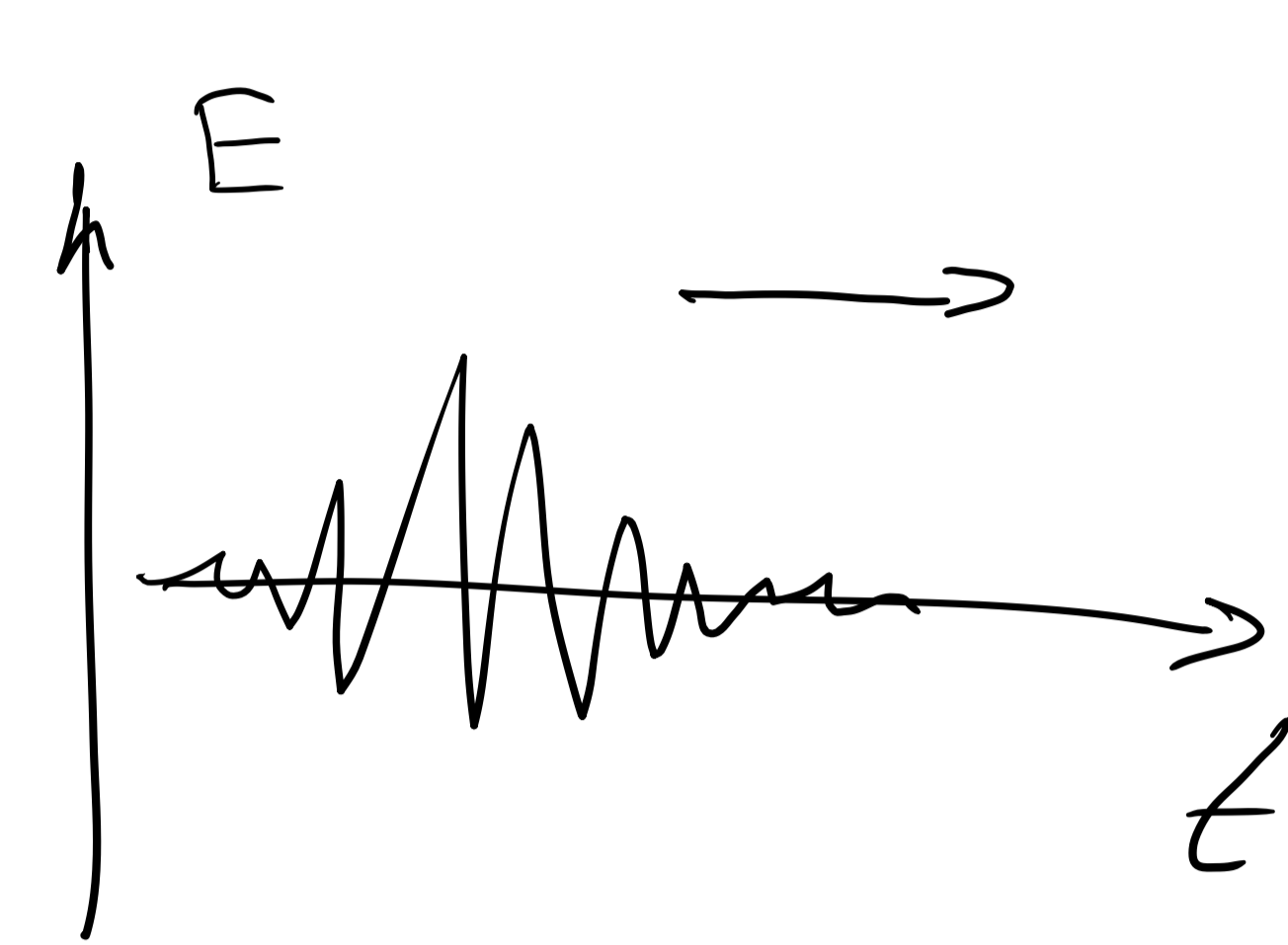
How can it be?

We need to answer what is it the ultimate speed limit?

Ultimate speed limit is the maximum speed we can transfer the signal.

At the same time, what is it the signal?

It is finite wave, which is modulated. However, we know, that if the wave is finite it is not monochromatic.



From Fourier transformation it is superposition of many monochromatic components.

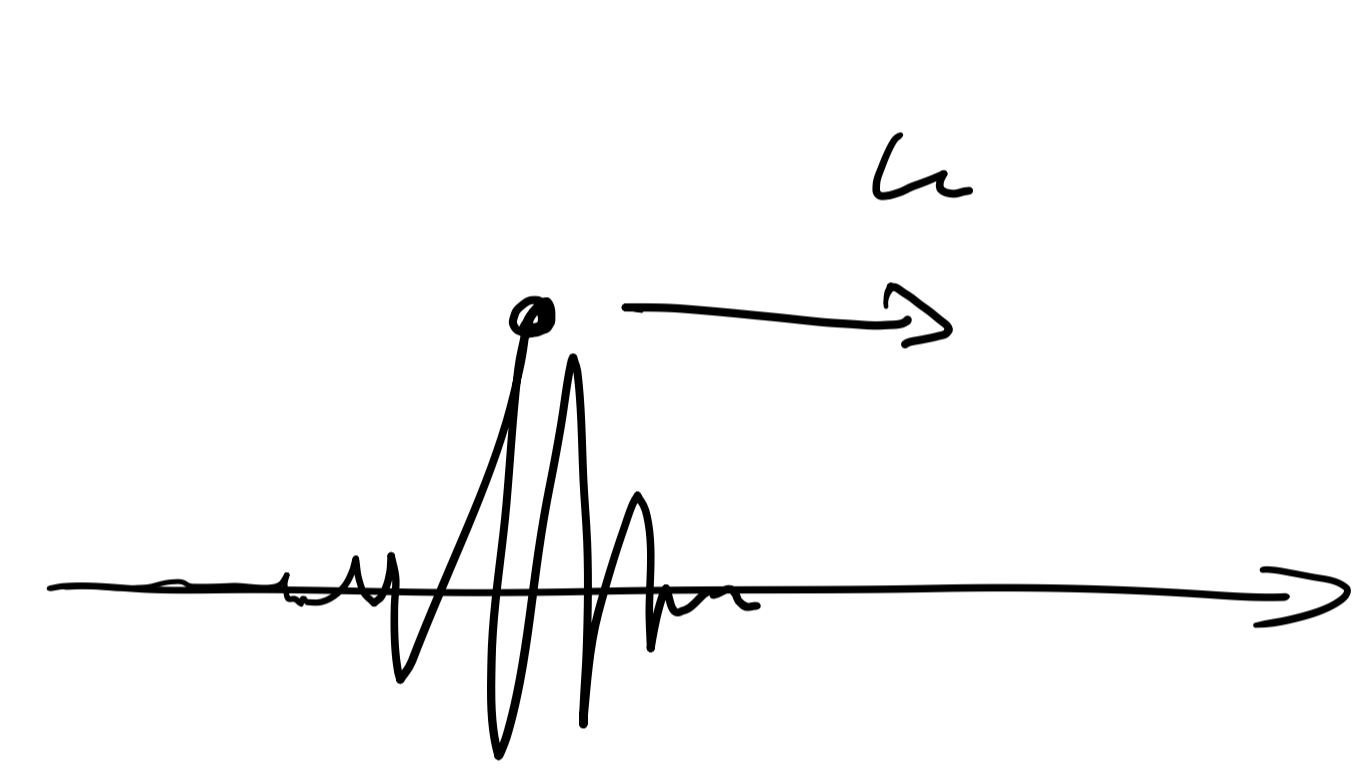
But since there is dispersion in the medium, each component will have its own speed v .

Here is the question:

What we call the speed of such signal transfer?

We can say that the whole packet of waves (group of waves) is moving with some speed u . We will call this speed group speed.

For example, here is signal:



We can look at the maximum of this signal and observe how it moves in space.

In other words, we will look at the surfaces of equal amplitudes.

The term surface of equal phase do not have physical meaning for wave packet.

For simplicity, we will take simple packet that consist of two waves:

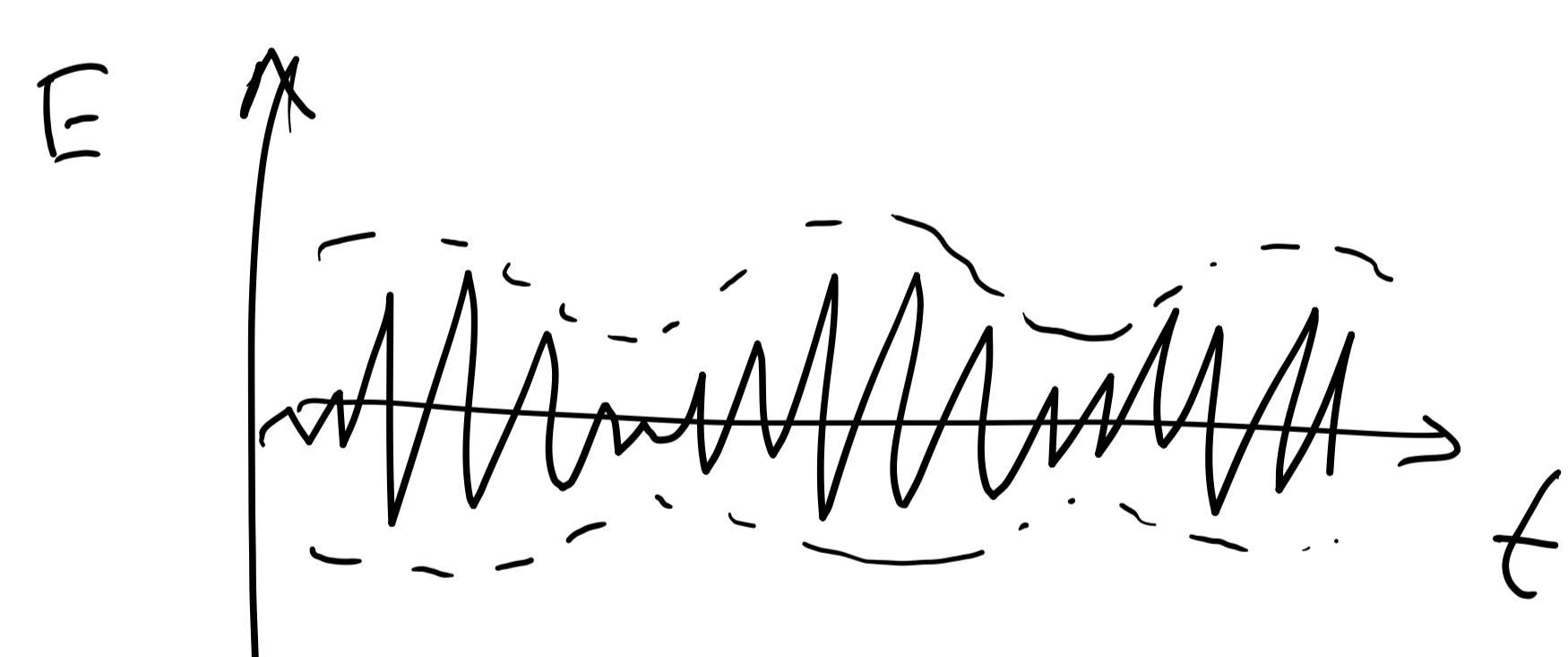
$$\begin{cases} E_1 = E_0 \cos(\omega t - k z) \\ E_2 = E_0 \cos[(\omega + \Delta\omega)t - (k + \Delta k)z] \end{cases}$$

$$\Delta\omega \ll \omega$$

$$\Delta k \ll k$$

$$E = E_1 + E_2 = 2E_0 \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}z\right) \cos(\omega t - k z)$$

$$\frac{2\pi}{\Delta\omega} \gg \frac{2\pi}{\omega}, \text{ thus one will observe beating:}$$



We will call slowly changing component an amplitude:

$$A = 2E_0 \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}z\right)$$

The surface of equal amplitude is:

$$\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}z = \text{const}$$

If we differentiate:

$$\Delta\omega dt - \Delta k dz = 0$$

Hence

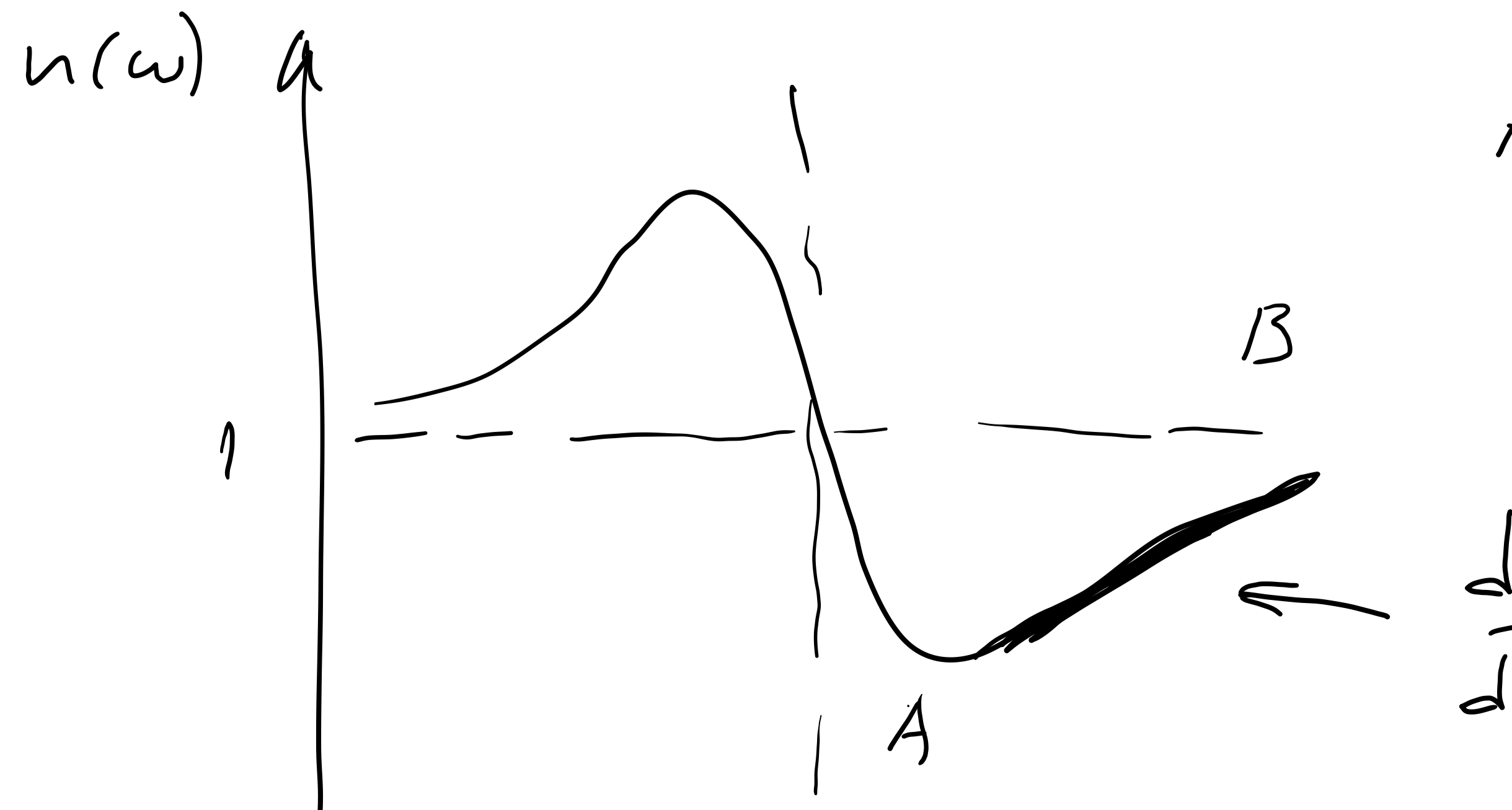
$$u = \frac{dz}{dt} = \frac{d\omega}{dk} \quad \omega = vk$$

$$u = \frac{d\omega}{dk} = \frac{vdk + kdv}{dk} = v + k \frac{dv}{dk} = v - \lambda \frac{dv}{d\lambda}$$

$$\text{Since } k = \frac{2\pi}{\lambda} \Rightarrow \frac{dk}{k} = -\frac{d\lambda}{\lambda}$$

$$\boxed{u = v - \lambda \frac{dv}{d\lambda}} \quad \text{Rayleigh formula}$$

This formula actually explains the previous paradox.



AB: $v > c$
but $\frac{dv}{d\lambda} > 0$

$\frac{dv}{d\lambda} > 0 \Rightarrow u < c$.